VOLUME 78 SEPARATE No. D-53

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

APRIL, 1952

AMERICAN SOCIETY OF CIVIL ENGINEERS FOUNDED 1852

DISCUSSION OF WEDGE-BEAM FRAMING

(Published in January, 1951)

By G. C. Ernst, J. J. Hromadik, and G. R. Swihart; Herbert A. Sawyer, Jr.; J. J. Polivka; and Arsham Amirikian

STRUCTURAL DIVISION

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

GUIDEPOST FOR TECHNICAL READERS

"Proceedings-Separates" of value or significance to readers in various fields are here listed, for convenience, in terms of the Society's Technical Divisions. Where there seems to be an overlapping of interest between Divisions, the same Separate number may appear under more than one item.

Technical Division	*Proceedings-Separate Number*
Air Transport	42, 43, 48, 52, 60, 93, 94, 95, 100, 103, 104, 108, 121 (Discussion: D-XXVIII, D-7, D-16, D-18, D-23, D-43)
City Planning	58, 60, 62, 64, 93, 94. 99, 101, 104, 105, 115 (Discussion: D-16, D-23, D-43, D-60, D-62, D-65)
Construction	43, 50, 55, 71, 92, 94, 103, 108, 109, 113, 117, 121, 126 (Discussion: D-3, D-8, D-17, D-23, D-36, D-40, D-71)
Engineering Mechanics	41, 49, 51, 54, 56, 59, 61, 88, 89, 96, 116, 122, 124, 125, 126 (Discussion: D-5, D-XXIII, D-XXV, D-18, D-24, D-33, D-34 D-49, D-54, D-61)
Highway	43, 44, 48, 58, 70, 100, 105, 108, 113, 120, 121 (Discussion: D-XXVIII, D-7, D-13, D-16, D-23, D-60)
Hydraulics	50, 55, 56, 57, 70, 71, 78, 79, 80, 83, 86, 92, 96, 106, 107, 110, 111, 112, 113, 116, 120, 123 (Discussion: D-XXVII, D-9, D-11, D-19, D-28, D-29, D-56, D-70, D-71)
Irrigation and Drainage	46, 47, 48, 55, 56, 57, 67, 70, 71, 87, 88, 90, 91, 96, 97, 98, 99, 102, 106, 109, 110, 111, 112, 114, 117, 118, 120 (Discussion: D-XXIII, D-3, D-7, D-11, D-17, D-19, D-25-K, D-29, D-30, D-38, D-40, D-44, D-47, D-57, D 70, D-71)
Power	48, 55, 56, 69, 71, 88, 96, 103, 106, 109, 110, 117, 118, 120 (Discussion: D-XXIII, D-2, D-3, D-7, D-11, D-17, D-19, D-25-K, D-30, D-38, D-40, D-44, D-70, D-71)
Sanitary Engineering	55, 56, 87, 91, 96, 106, 111, 118 (Discussion: D-10, D-29, D-37, D-56, D-60, D-70)
Soil Mechanics and Foundations	43, 44, 48, 94, 102, 103, 106, 108, 109, 115 (Discussion: D-4, D-XXVIII, D-7, D-43, D-44, D-56)
Structural	42, 49, 51, 53, 54, 59, 61, 66, 89, 100, 103, 109, 113, 116, 117, 119, 121, 122, 123, 124, 125, 126 (Discussion: D-3, D-5, D-8, D-13, D-16, D-17, D-21, D-23, D-24, D-25-K, D-32, D-33, D-34, D-37, D-39, D-42, D-49, D-51, D-53, D-54, D-59, D-61, D-72)
Surveying and Mapping	50, 52, 55, 60, 63, 65, 68, 121 (Discussion: D-60, D-65)
Waterways	41, 44, 45, 50, 56, 57, 70, 71, 96, 107, 112, 113, 115, 120, 123 (Discussion: D-8, D-9, D-19, D-27, D-28, D-56, D-70, D-71)

A constant effort is made to supply technical material to Society members, over the entire range of possible interest. Insofar as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement. Those who are planning papers for submission to "Proceedings-Separates" will expedite Division and Committee action measurably by first studying the ASCE "Guide for Development of Proceedings-Separates" as to style, content, and format. For a copy of this Manual, address the Manager, Technical Publications, ASCE, 33 W. 39th Street, New York 18, N. Y.

The Society is not responsible for any statement made or opinion expressed in its publications

Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

DISCUSSION

G. C. Ernst,[3] M. ASCE, J. J. Hromadik,[4] and G. R. Swihart,[5] J. M. ASCE.—A novel adaptation of the "dummy" or "unit" load method of determining deflections is proposed for solving the redundant crown forces in a connected series of three-hinged frames. The statical determinateness of these three-hinged units results in simple expressions for the forces that produce moments in the several members. This is one of the evident advantages of the analysis; but a more important advantage is the possible economy of material obtained in shaping the members to fit the stress pattern advantageously. It may be expected, however, that the unit cost of such construction will be considerably higher than for normal types until contractors become accustomed to the differences in details.

In the case of precast units of reinforced concrete the writers feel that the Considère hinge will have an advantage over the Mesnager hinge with regard to assembly, although either could be used. The advantage gained in using the Considère hinge lies in the fact that the bars need not cross, the steel may be welded, and a covering can be cast more readily after the subassemblies are in place, as shown in Fig. 16. The resulting appearance need not be different

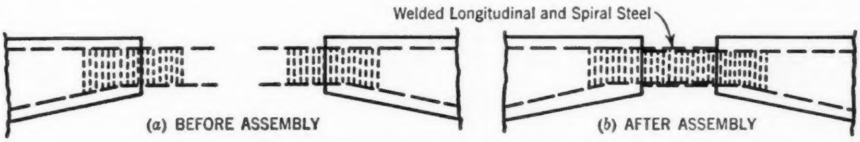


Fig. 16.—Assembly of the Considère Hinge

from that obtained with a Mesnager hinge.

Rotational resistance of such hinges, although substantially greater than the Mesnager hinge, is sufficiently low to permit an assumption of a hinged condition for the angular rotations met in practice. Design charts have been prepared for Considère hinges, based upon the use of a limiting deformation that will prevent unsightly cracking.[6]

Furthermore, the use of the flexible connection or joint of limited rigidity as proposed in Part I under "Connections" should logically raise a question as to the additional economy possible by the use of sections of predetermined capacity. It is just as logical to use a section of predetermined capacity, having a magnitude of resistance other than virtually zero, as it is to use the Mesnager or Considère articulations, and there is neither more nor less likeli-

Note.—This paper by Arsham Amirikian was published in January, 1951, as *Proceedings-Separate No. 53*. The numbering of footnotes, tables, equations, and illustrations in this Separate is a continuation of the consecutive numbering used in the original paper.

[3] Cons. Engr., Dunmire & Ernst, Lincoln, Nebr., and Prof. of Civ. Eng., Univ. of Nebraska, Lincoln, Nebr.

[4] Instr., Civ. Eng. Dept., Univ. of Nebraska, Lincoln, Nebr.

[5] Instr., Civ. Eng. Dept., Univ. of Nebraska, Lincoln, Nebr.

[6] "Tests of Considère Hinges Under Direct Stress, Bending, and Shear," by G. C. Ernst, *Proceedings*, A. C. I., Vol. 36, 1940, p. 49.

hood of exceeding the elastic properties to a detrimental degree in either case. Obviously, such an extension would necessitate a careful development of procedure.

A significant contribution has been made by the author in presenting a practical procedure for the proposed method of analysis. However, for those situations in which it would be desirable to vary the dimensions of the members or for occasions when symmetry of load within a bay does not occur, the substitution in the equations for the k-value (a relatively minor operation) is likely to deter many from taking full advantage of such framing. Although it is true that conditions of load symmetry and equal amounts of taper for the members remove the need to compute values of k, it is not likely that such a condi-

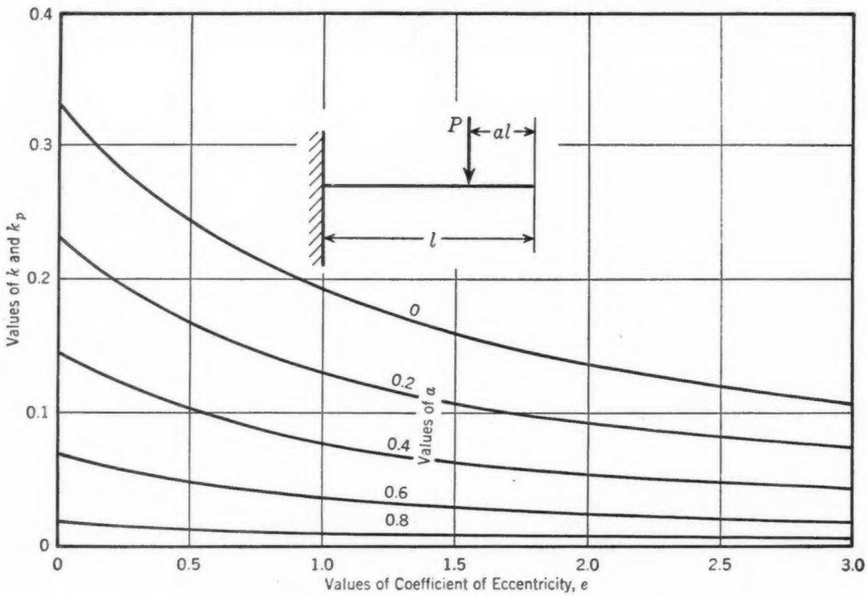
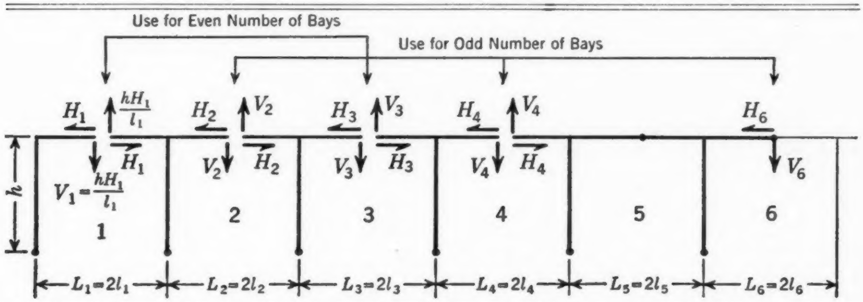


FIG. 17.—GRAPH FOR SOLUTION OF DEFLECTION FORMULA TAPERED FLANGE BEAM

tion can always be used. The equations of Tables 1 through 5 may be made more useful by placing them in chart form, such as the example presented in Fig. 17 prepared for a section of Table 1. Such charts could be readily plotted for the remaining parts of Table 1 and for Tables 2 through 5 and would markedly increase the value of the author's proposed method.

The writers feel that a valuable further simplification of the tables of coefficients for the contributions of members to the deflection of a joint may be developed. The coefficients for H and V for values of ΔH and ΔV do not change with variations in applied loading, and the coefficients shown in Table 7 may be used for the end bays of frames with an odd number of bays. Therefore, it is possible to combine parts of Tables 7, 8, and 9 into a single table as shown in Table 10.

TABLE 10.—Contribution of Members to Deflection of Joints in End or Interior Bays of Rectangular Frames for an Even or Odd Number of Bays

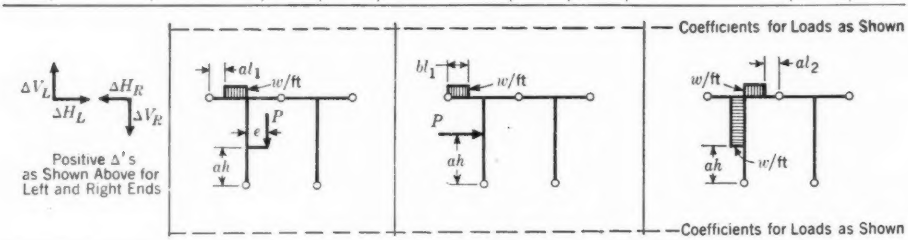


Applicable Values of Δ						Coefficients H and V for ΔH_1 in End Bay of Even Number of Bays			Coefficients H and V for Δ_{H2} and Δ_{V2} in End Bays of Odd Number of Days					
										$h\,H_2$	$h\,H_4$		$l_2\,V_2$	$l_4\,V_4$
									Coefficients H and V for Δ_{H3} and Δ_{V3} in Interior Bays of Even Number of Bays					
									$h\,H_1$	$h\,H_3$	$h\,H_5$	$l_1\,H_1$	$l_3\,V_3$	$l_5\,V_5$
Δ_{H1}, Δ_{H2}, Δ_{V2}		Δ_{H3}, Δ_{V3}		Δ_{H4}, Δ_{V4}					Coefficients H and V for Δ_{H4} and Δ_{V4} in Interior Bays of Odd Number of Bays					
Member (1)	C[a] (2)	Member (3)	C (4)	Member (5)	C (6)	$h\,H_1$ (7)	$h\,H_3$ (8)	$l_3\,V_3$ (9)	$h\,H_2$ (10)	$h\,H_4$ (11)	$h\,H_6$ (12)	$l_2\,V_2$ (13)	$l_4\,V_4$ (14)	$l_6\,V_6$ (15)
(a) Coefficients for Horizontal Deflection														
Beam:		Beam:		Beam:										
$1_{L,R}$	C_1	$2_{L,R}$	C_2	$3_{L,R}$	C_3	2			−½	½		−½	−½	
$2_{L,R}$	C_2	$3_{L,R}$	C_3	$4_{L,R}$	C_4	2	−1	−1						
$3_{L,R}$	C_3	$4_{L,R}$	C_4	$5_{L,R}$	C_5					½	−½		½	½
Column:		Column:		Column:										
0–1	$C_{0,1}$	1–2	$C_{1,2}$	2–3	$C_{2,3}$	1			−¼	¼		¼	−¼	
1–2	$C_{1,2}$	2–3	$C_{2,3}$	3–4	$C_{3,4}$				−¼	¼		−¼	¼	
2–3	$C_{2,3}$	3–4	$C_{3,4}$	4–5	$C_{4,5}$	1	−½	−½		¼	−¼		−¼	¼
3–4	$C_{3,4}$	4–5	$C_{4,5}$	5–6	$C_{5,6}$					¼	−¼		¼	−¼
(b) Coefficients for Vertical Deflection														
Beam:		Beam:		Beam:										
$1_{L,R}$	C'_1	$2_{L,R}$	C'_2	$3_{L,R}$	C'_3				½	−½		½	½	
$2_{L,R}$	C'_2	$3_{L,R}$	C'_3	$4_{L,R}$	C'_4								2	
$3_{L,R}$	C'_3	$4_{L,R}$	C'_4	$5_{L,R}$	C'_5					½	−½		½	½
Column:		Column:		Column:										
0–1	$C'_{0,1}$	1–2	$C'_{1,2}$	2–3	$C'_{2,3}$				¼	−¼		−¼	¼	
1–2	$C'_{1,2}$	2–3	$C'_{2,3}$	3–4	$C'_{3,4}$				−¼	¼		−¼	¼	
2–3	$C'_{2,3}$	3–4	$C'_{3,4}$	4–5	$C'_{4,5}$					−¼	¼		¼	−¼
3–4	$C'_{3,4}$	4–5	$C'_{4,5}$	5–6	$C'_{5,6}$					¼	−¼		¼	−¼
(c) Totals[b]														
Δ_{H1}						6	−1½	½						
Δ_{H2}										2	−1		0	½
Δ_{H3} and Δ_{H4}									−1	2	−1	−½	0	½
Δ_{V2}										0	−½		4	0
Δ_{V3} and Δ_{V4}									½	0	−½	0	4	0

[a] $C = k\,h\,\frac{l}{I}$; $C' = k\,l_n\,\frac{l}{I}$. [b] Use for same-shaped members, equal spans, and $h = l$ only.

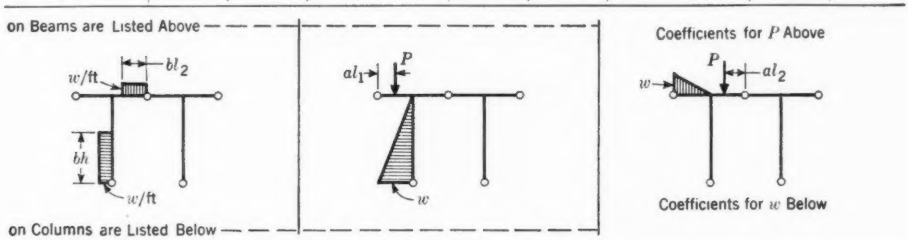
TABLE 11.—CONTRIBUTION OF MEMBERS TO DEFLECTION

Member	Common dimensional multiplier	Load multiplier k	Δ_L	Δ_R	Load multiplier k	Δ_L	Δ_R	Load multiplier k	Δ_L	Δ_R
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
(a) LOAD COEFFICIENTS FOR ΔH FOR BEAMS AND COLUMNS,										
1			0	0		0	0		0	0
2_L	$h\,l_2/I_{2L}$	$(1-a)^2 l^2_1\, w\, k$	$-\frac{1}{8}$	$+\frac{1}{8}$	$\left(1-\frac{b}{2}\right) b\, l^2_1\, w\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$\begin{cases}(1-a)^2 l^2_2\, w\, k \\ l^2_2\, w\, k_{w3}\end{cases}$	$+\frac{1}{8}$ $-\frac{1}{2}$	$-\frac{1}{8}$ $+\frac{1}{2}$
2_R	$h\,l/I_{2R}$	$(1-a)^2 l^2_1\, w\, k$	$-\frac{1}{8}$	$+\frac{1}{8}$	$\left(1-\frac{b}{2}\right) b\, l^2_1\, w\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)^2 l^2_2\, w\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$
3			0	0		0	0		0	0
1-2	$h^2/I_{1,2}$	$(1-a)^2 l^2_1\, w\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$\left(1-\frac{b}{2}\right) b\, l^2_1\, w\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)^2 l^2_2\, w\, k$	$-\frac{1}{8}$	$+\frac{1}{8}$
2-3	$h^2/I_{2,3}$	$(1-a)^2 l^2_1\, w\, k$	$-\frac{1}{8}$	$+\frac{1}{8}$	$\left(1-\frac{b}{2}\right) b\, l^2_1\, w\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)^2 l^2_2\, w\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$
(b) LOAD COEFFICIENTS FOR ΔV FOR BEAMS AND COLUMNS,										
1	l_1/I_1	$l^3_1\, w\, k_{w3}$	-1	0	$l^2_1\, w\, k_{w4}$	-1	0		0	0
2_L	$l_2/I_2\Delta$	$(1-a)^2 l^2_1\, w\, k$	$-\frac{1}{8}\, l_1$	$-\frac{1}{8}\, l_3$	$\left(1-\frac{b}{2}\right) b\, l^2_1\, w\, k$	$-\frac{1}{4}\, l_1$	$-\frac{1}{4}\, l_3$	$\begin{cases}(1-a)^2 l^2_2\, w\, k \\ l^2_2\, w\, k_{w3}\end{cases}$	$+\frac{1}{8}\, l_1$ $-\frac{1}{2}\, l_1$	$+\frac{1}{8}\, l_3$ $-\frac{1}{2}\, l_3$
2_R	l_2/I_{2R}	$(1-a)^2 l^2_1\, w\, k$	$-\frac{1}{8}\, l_1$	$-\frac{1}{8}\, l_3$	$\left(1-\frac{b}{2}\right) b\, l^2_1\, w\, k$	$-\frac{1}{4}\, l_1$	$-\frac{1}{4}\, l_3$	$(1-a)^2 l^2_2\, w\, k$	$+\frac{1}{8}\, l_1$	$+\frac{1}{8}\, l_3$
3			0	0		0	0		0	0
1-2	$h/I_{1,2}$	$(1-a)^2 l^2_1\, w\, k$	$-\frac{1}{8}\, l_1$	$+\frac{1}{8}\, l_3$	$\left(1-\frac{b}{2}\right) b\, l^2_1\, w\, k$	$-\frac{1}{4}\, l_1$	$+\frac{1}{4}\, l_3$	$(1-a)^2 l^2_2\, w\, k$	$+\frac{1}{8}\, l_1$	$-\frac{1}{8}\, l_3$
2-3	$h/I_{2,3}$	$(1-a)^2 l^2_1\, w\, k$	$-\frac{1}{8}\, l_1$	$+\frac{1}{8}\, l_3$	$\left(1-\frac{b}{2}\right) b\, l^2_1\, w\, k$	$-\frac{1}{4}\, l_1$	$+\frac{1}{4}\, l_3$	$(1-a)^2 l^2_2\, w\, k$	$+\frac{1}{8}\, l_1$	$-\frac{1}{8}\, l_3$



Member	Common dimensional multiplier	Load multiplier k	Δ_L	Δ_R	Load multiplier k	Δ_L	Δ_R	Load multiplier k	Δ_L	Δ_R
(c) LOAD COEFFICIENTS FOR ΔH FOR BEAMS AND COLUMNS,										
1			0	0		0	0		0	0
2_L	$h\,l_2/I_{2L}$	$P_y\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$P\, a\, h\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a^2)\, w\, h^2\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$
2_R	$h\,l_2/I_{2R}$	$P_y\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$P\, a\, h\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a^2)\, w\, h^2\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$
3			0	0		0	0		0	0
1-2	$h^2/I_{1,2}$	$P_y\, k_m$	$\begin{cases}-\frac{1}{4} \\ +\frac{1}{2}\end{cases}$	$+\frac{1}{4}$ $-\frac{1}{2}$	$\left(1-\frac{a}{2}\right) P\, h\, k$ $P\, h\, k_p$	$+\frac{1}{2}$ $-\frac{1}{2}$	$-\frac{1}{2}$ $+\frac{1}{2}$	$\begin{cases}(1-a)(3-a)\, w\, h^2\, k \\ w\, h^2\, k_{w3}\end{cases}$	$+\frac{1}{8}$ $-\frac{1}{2}$	$-\frac{1}{8}$ $+\frac{1}{2}$
2-3	$h^2/I_{2,3}$	$P_y\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$P\, a\, h\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a^2)\, w\, h^2\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$
(d) LOAD COEFFICIENTS FOR ΔV FOR BEAMS AND COLUMNS,										
1	l_1/I_1		0	0		0	0		0	0
2_L	l_2/I_{2L}	$P_y\, k$	$+\frac{1}{4}\, l_1$	$+\frac{1}{4}\, l_3$	$P\, a\, h\, k$	$+\frac{1}{4}\, l_1$	$+\frac{1}{4}\, l_3$	$(1-a^2)\, w\, h^2\, k$	$+\frac{1}{8}\, l_1$	$+\frac{1}{8}\, l_3$
2_R	l_2/I_{2R}	$P_y\, k$	$+\frac{1}{4}\, l_1$	$+\frac{1}{4}\, l_3$	$P\, a\, h\, k$	$+\frac{1}{4}\, l_1$	$+\frac{1}{4}\, l_3$	$(1-a^2)\, w\, h^2\, k$	$+\frac{1}{8}\, l_1$	$+\frac{1}{8}\, l_3$
3			0	0		0	0		0	0
1-2	$h/I_{1,2}$	$\begin{cases}P_y\, k \\ P_y\, k_m\end{cases}$	$+\frac{1}{4}\, l_1$ $-\frac{1}{2}\, l_1$	$-\frac{1}{4}\, l_3$ $+\frac{1}{2}\, l_3$	$\left(1-\frac{a}{2}\right) P\, h\, k$ $P\, h\, k_p$	$-\frac{1}{2}\, l_1$ $+\frac{1}{2}\, l_1$	$+\frac{1}{2}\, l_3$ $-\frac{1}{2}\, l_3$	$(1-a)(3-a)\, w\, h^2\, k$ $w\, h^2\, k_{w3}$	$-\frac{1}{8}\, l_1$ $+\frac{1}{2}\, l_1$	$+\frac{1}{8}\, l_3$ $-\frac{1}{2}\, l_3$
2-3	$h/I_{2,3}$	$P_y\, k$	$+\frac{1}{4}\, l_1$	$-\frac{1}{4}\, l_3$	$P\, a\, h\, k$	$+\frac{1}{4}\, l_1$	$-\frac{1}{4}\, l_3$	$(1-a^2)\, w\, h^2\, k$	$+\frac{1}{8}\, l_1$	$-\frac{1}{8}\, l_3$

Load multiplier k (12)	Δ_L (13)	Δ_R (14)	Load multiplier k (15)	Δ_L (16)	Δ_R (17)	Load multiplier k (18)	Δ_L (19)	Δ_R (20)
WITH LOADING ON BEAMS ONLY AS SHOWN BELOW								
	0	0		0	0		0	0
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$
$l_2^2\, w\, k_{w4}$	$-\frac{1}{2}$	$+\frac{1}{2}$				$l_2\, P\, k_p$	$-\frac{1}{2}$	$+\frac{1}{2}$
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$
	0	0		0	0		0	0
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$-\frac{1}{4}$	$+\frac{1}{4}$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4}$	$-\frac{1}{4}$
WITH LOADING ON BEAMS ONLY AS SHOWN BELOW								
	0	0	$l_1^2\, P\, k_p$	-1	0		0	0
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$
$l_2^2\, w\, k_{w4}$	$-\frac{1}{2} l_1$	$-\frac{1}{2} l_3$				$l_2\, P\, k_p$	$-\frac{1}{2} l_1$	$-\frac{1}{2} l_3$
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4} l_1$	$+\frac{1}{4} l_3$
	0	0		0	0		0	0
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$
$\left(1-\frac{b}{2}\right) b\, l_2^2\, w\, k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$	$(1-a)\, l_1\, P\, k$	$-\frac{1}{4} l_1$	$+\frac{1}{4} l_3$	$(1-a)\, l_2\, P\, k$	$+\frac{1}{4} l_1$	$-\frac{1}{4} l_3$



Load multiplier k (12)	Δ_L (13)	Δ_R (14)	Load multiplier k (15)	Δ_L (16)	Δ_R (17)	Load multiplier k (18)	Δ_L (19)	Δ_R (20)
WITH LOADING ON COLUMNS ONLY AS SHOWN ABOVE								
	0	0		0	0		0	0
$w\,(b\,h)^2\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$w\, h^2\, k$	$+\frac{1}{24}$	$-\frac{1}{24}$	$w\, l_1^2\, k$	$-\frac{1}{12}$	$+\frac{1}{12}$
$w\,(b\,h)^2\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$w\, h^2\, k$	$+\frac{1}{24}$	$-\frac{1}{24}$	$w\, l_1^2\, k$	$-\frac{1}{12}$	$+\frac{1}{12}$
	0	0		0	0		0	0
$(4-b)\, w\, b\, h^2\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$w\, h^2\, k$	$+\frac{5}{24}$	$-\frac{5}{24}$	$w\, l_1^2\, k$	$+\frac{1}{12}$	$-\frac{1}{12}$
$w\, h^2\, k_{w4}$	$-\frac{1}{2}$	$+\frac{1}{2}$	$w\, h^2\, k_{w2}$	$-\frac{1}{2}$	$+\frac{1}{2}$			
$w\,(b\,h)^2\, k$	$+\frac{1}{8}$	$-\frac{1}{8}$	$w\, h^2\, k$	$+\frac{1}{24}$	$-\frac{1}{24}$	$w\, l_1^2\, k$	$-\frac{1}{12}$	$+\frac{1}{12}$
WITH LOADING ON COLUMNS ONLY AS SHOWN ABOVE								
	0	0		0	0	$w\, l_1^2\, k_{w2}$	-1	0
$w\,(b\,h)^2\, k$	$+\frac{1}{8} l_1$	$+\frac{1}{8} l_3$	$w\, h^2\, k$	$+\frac{1}{24} l_1$	$+\frac{1}{24} l_3$	$w\, l_1^2\, k$	$-\frac{1}{12} l_1$	$-\frac{1}{12} l_3$
$w\,(b\,h)^2\, k$	$+\frac{1}{8} l_1$	$+\frac{1}{8} l_3$	$w\, h^2\, k$	$+\frac{1}{24} l_1$	$+\frac{1}{24} l_3$	$w\, l_1^2\, k$	$-\frac{1}{12} l_1$	$-\frac{1}{12} l_3$
	0	0		0	0		0	0
$(4-b)\, w\, b\, h^2\, k$	$-\frac{1}{8} l_1$	$+\frac{1}{8} l_3$	$w\, h^2\, k_{w2}$	$-\frac{5}{24} l_1$	$+\frac{5}{24} l_3$	$w\, l_1^2\, k$	$-\frac{1}{12} l_1$	$+\frac{1}{12} l_3$
$w\, h^2\, k_{w4}$	$+\frac{1}{2} l_1$	$-\frac{1}{2} l_3$	$w\, h^2\, k$	$+\frac{1}{2} l_1$	$-\frac{1}{2} l_3$			
$w\,(b\,h)^2\, k$	$+\frac{1}{8} l_1$	$-\frac{1}{8} l_3$	$w\, h^2\, k$	$+\frac{1}{24} l_1$	$-\frac{1}{24} l_3$	$w\, l_1^2\, k$	$-\frac{1}{12} l_1$	$+\frac{1}{12} l_3$

It is also evident that the load coefficients shown in Tables 7, 8, and 9 are applicable only to the type of loading shown and applicable only if symmetry of loading within a bay is maintained, although all bays need not be loaded. The writers are of the opinion that data in the form of Table 11 are more useful and, of course, cover a broader range of loading conditions. An arrangement similar to Table 11 could also be developed for the end bays of frames with an even number of bays. When used in conjunction with charts, such as those proposed herein for Tables 1 through 5, Tables 10 and 11 will provide an effective design tool for the author's proposed method of analysis.

TABLE 12.—EXACT METHOD OF SOLUTION FOR THE SIMULTANEOUS EQUATIONS FROM THE FRAME IN FIG. 13(a)

Equation No.	Joint No.	H_2	V_2	H_4	V_4	H_6	V_6	Constant in terms of wl
				(a) TABULATION OF COEFFICIENTS OF ORIGINAL EQS. 22				
1	2	+2		−1	+0.5			0
2			+4	−0.5				−0.5
3	4	−1	−0.5	+2		−1	+0.5	0
4		+0.5			+4	−0.5		+0.5
5	6			−1	−0.5	+2		0
6				+0.5			+4	0
				(b) EXACT SOLUTION BY SUCCESSIVE ELIMINATION OF UNKNOWNS				
1′		+1.0		−0.5	+0.25			0
4′		+1.0			+8.00	−1.0		+1.0
3′		−1.0	−0.5	+2.0		−1.0	+0.5	0
7				+0.5	+7.75	−1.0		+1.0
8			−0.5	+1.5	+0.25	−1.0	+0.5	0
5′				−0.5	−0.25	+1.0		0
9			−0.5	+1.0	−7.50		+0.5	−1.0
10					+7.50			+1.0
6′				+1.0			+8.0	0
9′			−0.5	+1.0			+0.5	0
2′			+8.0	−1.0				−1.0
11			+8.0				+8.0	−1.0
12			+7.5				+0.5	−1.0
11′			+1.0				+1.0	−0.125
12′			+15.0				+1.0	−2.000
13			+14.0					−1.875
Final Values H and V		−0.0690	−0.1339	−0.0714	0.1333	−0.0024	0.0089	

It is not clear to the writers why the method of summation of successive increments is proposed for the solution of the simultaneous equations. The equations take the form of secondary stress equations and in some respects might well be considered as such. The method of successive approximations, frequently used in the solution of secondary stress equations, could be adapted to the wedge-beam framing equations more advantageously than the summation of successive increments. Comparison of the results obtained by an exact solu-

tion of Eqs. 22 by successive elimination of unknowns with those obtained by successive approximations, as well as the author's results, are shown in Tables 12 and 13. The method of successive approximations removes the need of adding increments to obtain the final answer and will converge more rapidly on the true values for the equations shown. Furthermore, convergence is most rapid if substitution is made as soon as approximate values are obtained, as may be observed at the bottom of Table 13.

The writers feel that the author's proposed method of tabular coefficients is of greatest advantage when it is necessary (as is usually the case) to carry through solutions for a given frame for different loadings. Maximum stresses resulting from various combinations of loads—such as wind load, crane load,

TABLE 13.—SOLUTION BY SUCCESSIVE APPROXIMATIONS

Quantity	Approximations:						Actual values	Author's values
	First	Second	Third	Fourth	Fifth	Sixth		
(a) ASSUMING EQUAL VALUES FOR THE UNKNOWNS IN DETERMINING THE FIRST APPROXIMATION								
H_2	0	−0.0313	−0.0492	−0.0489	−0.0590	−0.0627	−0.0690	−0.0577
V_2	−0.1429	−0.1250	−0.1295	−0.1289	−0.1314	−0.1324	−0.1339	−0.1311
H_4	0	−0.0358	−0.0313	−0.0516	−0.0489	−0.0588	−0.0714	−0.0587
V_4	+0.1250	+0.1250	+0.1328	+0.1328	+0.1333	+0.1333	+0.1333	+0.1333
H_6	0	+0.0313	+0.0133	+0.0176	+0.0074	+0.0040	−0.0024	+0.0088
V_6	0	0	+0.0045	+0.0039	+0.0065	+0.0076	+0.0089	+0.0061
(b) ZERO ASSUMED FOR ALL, EXCEPTING THE PRINCIPAL UNKNOWN, UNTIL VALUES ARE OBTAINED FOR IMMEDIATE SUBSTITUTION IN SUBSEQUENT OPERATIONS								
H_2	0	−0.0469	−0.0577	−0.0627	−0.0655	−0.0671	−0.0690	−0.0577
V_2	−0.1250	−0.1289	−0.1311	−0.1323	−0.1331	−0.1335	−0.1339	−0.1311
H_4	−0.0313	−0.0489	−0.0588	−0.0644	−0.0675	−0.0693	−0.0714	−0.0587
V_4	0.1250	−0.1328	0.1333	0.1333	0.1333	0.1333	0.1333	0.1333
H_6	0.0156	0.0087	0.0039	0.0011	−0.0004	−0.0013	−0.0024	0.0089
V_6	0.0039	0.0061	0.0074	0.0081	0.0084	0.0087	0.0089	0.0061

conveyor system load, and roof load—may be readily obtained after the H- and V-values are known. Considerable saving may be noted in the fact that the coefficients for H and V do not change for the various loadings, and the equations may be solved in the same routine for all constants. The charts and Tables 10 and 11 proved distinctly valuable in setting up the equations for such an analysis.

Solution by successive elimination of unknowns is advocated, although successive approximations would be advantageous in the case of a large number of bays.

The author is to be congratualted upon his approach to the problem, and it is hoped that further simplifications will be forthcoming.

HERBERT A. SAWYER, JR.,[7] A. M. ASCE.—The primary reason advanced (in the paragraph headed "Range of Application") for the introduction and use of wedge-beam framing is economy in material and construction costs.

[7] Associate Prof., Dept. of Civ. Eng., Univ. of Connecticut, Storrs, Conn.

Engineering practice has long demonstrated that, for given loads and spans, a beam with varying strength along its length will weigh less than a beam of constant strength; and, if the added fabrication cost and change in erection cost of this variable strength beam are not greater than its material savings it is certainly more economical.

Another related, yet more elusive, source of economy of material is to change the effective span lengths of beams by a change in articulation or type of support, in effect changing the moment diagrams. Possibilities for change in articulation range from the classic post-and-lintel construction, with hinges over the columns, to the only slightly less classic cantilever construction, with hinges at midspan. A compromise between these extremes is the more modern continuous construction, with "hinges" or points of contraflexure usually within the second and fifth sixths of the span length. This paper considers only cantilever construction because (as stated in the "Introduction") the arrangement of two simple cantilevers "* * * yields appreciable savings in weight * * *"

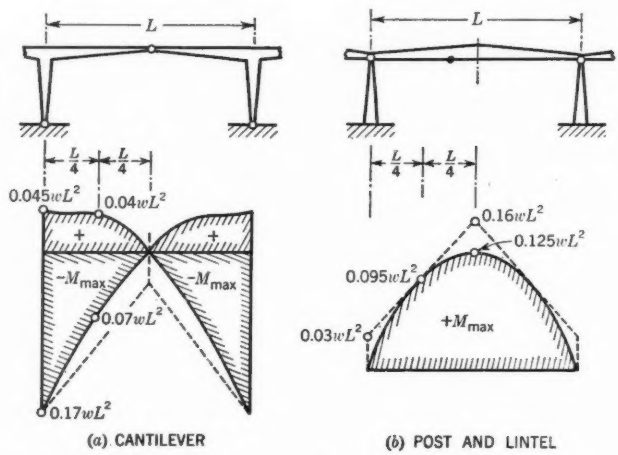


FIG. 18.—COMPARISON OF TAPERED BEAM REQUIREMENTS OF FRAMING

The writer realizes it is dangerous to attempt to generalize on the relative economy of the various types of articulation on the basis of a single analysis, because no single analysis can account for all the many types of loads, materials, and stresses. However, one analysis offers some evidence, so in Fig. 18 an attempt has been made to compare the beam requirements for post-and-lintel and for cantilever construction. The curves of maximum moment are for a moving uniform gravity load equal to w per unit length, on a frame of many bays, and the half-span cantilever beam is assumed to have the same stiffness as the column. As can be seen, the curves of maximum moment have been enveloped by a dotted line representing the maximum resisting moments of a tapered beam, showing graphically the minimum strength beam required to resist the maximum moments. The resisting moment of this beam was assumed to

vary linearly from a maximum down to a value of one fifth the maximum, corresponding quite closely to the type of strength variation of beams of the author's Tables 1, 3, and 5. Dead load moments have been neglected because they would have about the same effect on both types, although actually the small component of dead load moment from the weight of the beam itself would be somewhat less for the cantilever type since its weight is concentrated nearer the supports.

According to this analysis, cantilever construction requires a slightly heavier beam than post-and-lintel construction. However, other important factors, not apparent from Fig. 18, should be considered. The maximum shear occurs at the deepest section of the cantilever beam and at the shallowest section of the lintel beam. Thus, extra material could well be required to resist shear in the lintel beam, even though an important part of the shear is resisted by the nonparallel flanges. On the other hand, large gravity-load moments occur in the columns of the cantilever frame but not in the columns of the post-and-lintel frame. Also, unlike the lintel beam moments, the cantilever beam moments will be reversible (unless the dead load is improbably high), resulting in extra tensile steel in reinforced concrete.

A practical consideration affecting construction costs is that, in post-and-lintel construction, field joints can be at the hinges and the resulting pieces are convenient in shipping and erection. However, for cantilever construction the engineer is faced with the choice of shipping awkward **T**-segments or having a field splice near the point of maximum moment and shear.

A consideration of engineering effort (unfortunately not very important economically) would certainly favor the statically determinate post-and-lintel analysis over the statically indeterminate cantilever analysis described in the paper.

The conclusion of this analysis is that for the conditions investigated there is probably no significant difference in the amount of material required for post-and-lintel framing and cantilever framing using tapered beams. Therefore, the cantilever framing as described seems to have no important point of superiority over the more traditional type.

Possibly, the compromise between the two types here investigated—continuous construction—could be shown to be the most economical materially. The differences, however, are surely not great, and such an attempt is beyond the scope of this discussion.

If an analysis of cantilevered framing is required, it is interesting to note that the deflection formulas presented in Tables 1 to 5 may be used to solve any of the illustrative examples by the method of moment distribution devised by Hardy Cross,[8] Hon. M. ASCE. If moment distribution is used, the mathematics and simultaneous equations of the latter part of the paper are avoided.

To use moment distribution, the carry-over factors, fixed-end moments, and relative stiffnesses for all spans of a wedge-beam frame may be found using Tables 1 to 5 and Eqs. 28, 30, and 31 which follow. Moments may then be distributed, as described by Mr. Cross, to obtain the final moments.

[8] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 1.

Considering the typical wedge-beam span of Fig. 19 the carry-over factor from point C to point D (R_{CD}) is statically determinate:

$$R_{CD} = \frac{M_D}{M_C} = \frac{l_D}{l_C} \quad \ldots\ldots (28)$$

in which M_C is the moment at point C accompanying a rotation of support C, and M_D is the moment induced at point D from this rotation.

The fixed-end moment at point C from any loads on span CD (or M_{FC}) is equal to the moment at point C considering the member CA as a simple cantilever (M_{SC}) plus the moment at C from V_A, the shear at hinge A:

$$M_{FC} = M_{SC} + l_C V_A \quad \ldots\ldots (29)$$

The shear at point A may be found by the common method of removing the connecting hinge at point A and dividing the resulting difference in deflection of the two simple cantilevers at point A by the difference in deflection of the two cantilevers loaded only with opposite unit loads acting at point A. Replacing the shear by the quotient of these deflections, as obtained from Tables 1 to 5 and assuming that the values of $E\,I$ near the hinge on both sides are equal (as shown by Fig. 5), Eq. 29 becomes

$$M_{FC} = M_{SC} + \frac{m_C - m_D R^2_{CD}}{(k)_C + (k)_D R^3_{CD}} \quad \ldots\ldots (30)$$

in which k is the k (without subscript) of Tables 1 to 5, and m is equal to $k_p\,P\,l$, $k_m\,M$, or $k_w\,w\,l^2$, all as defined by Tables 1 to 5. For sign convention, both terms in the denominator are always positive; m is positive for downward deflection at point A of the simple cantilever, and M_{SC} and the resulting M_{FC} are both negative for a moment that produces tension in the upper fiber.

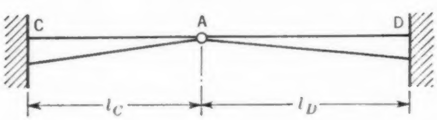


FIG. 19.—TYPICAL WEDGE-BEAM SPAN

The relative stiffness of member CD for rotation at point C (K_{CD}) or the moment at point C for a unit rotation at this point may easily be shown to be

$$K_{CD} = \frac{E\,I}{l_C\,[(k)_C + (k)_D R^3_{CD}]} \quad \ldots\ldots (31)$$

in which I is as defined in Tables 1 to 5.

For the column members of wedge-beam framing, l_D equals zero, and R_{CD} becomes zero in all the foregoing equations.

The author's illustrative examples involve spans symmetrically proportioned and symmetrically loaded about the hinges. The carry-over factors, fixed-end moments, and relative stiffnesses would be 1, M_{SC}, and $E\,I/(k\,l)$, respectively, for these spans, and 0, 0 and $E\,I/(k\,l)$, respectively, for the columns.

The analysis of two-dimensional frames in this paper neglects the restraints of the roofing or flooring and adjacent framing in the third dimension in pre-

venting sidesway. Moment distribution, on the other hand, assumes complete restraint from sidesway. This condition is nearer the actual action of framing in a building of many bays. For a building of a very few bays the effect of sidesway should usually be checked. Using moment distribution, this correction need only be made for one loading for a given frame, the correction for any other loading being found by direct proportion from the first determination.

Solutions for types of framing with beams whose axes depart significantly from the horizontal, such as those of Fig. 4, were not included in the paper. For these types, the sidesway of each joint should usually be accounted for in the analysis, as is done by the author's method of simultaneous equations. Using moment distribution, shears as well as moments must be distributed, as described in the paper by Mr. Cross and in the discussions of the paper. Tables 1 to 5 are helpful in determining the constants required by either method, for the frame types of Fig. 4(*a*) and Fig. 4(*d*) only. Constants must be derived for other frame types, such as those of Fig. 4(*b*) and Fig. 4(*c*).

J. J. Polivka,[9] M. ASCE.—When the great economy of precast concrete structures is finally recognized, this paper becomes especially valuable. The serious drawback of tedious analysis is considerably alleviated by the author's simplified solution, although the computation of the values of the factor (k) in Tables 1 to 5 requires considerable time and great accuracy. For this reason these factors for various shapes of tapered beams and various ratios of the moments of inertia at both ends of the beams can be tabulated. A simpler method of computing elastic deformations of any cross section and shape of a beam can be demonstrated[10] by reference to the special case (*a*) the three-bay frame with symmetrical framing and loading, shown in Fig. 9. The moment of the exterior corner (X) is considered to be redundant (Fig. 20). Equaling the deflections of the columns, the following equation determines X:

$$\Delta_L = G_3\, s_3\, d_3 \frac{X}{h} + G_1\, s_1\, d_1 \frac{X}{l_1} = \Delta_R = -\, G_1\, s_1\, d_1 \frac{X}{l_1} + \frac{(M - X)}{h} G_4\, s_4\, d_4 \qquad (32)$$

and

$$X = \frac{M\, G_4\, s_4\, d_4}{G_3\, s_3\, d_3 + \dfrac{2\, h}{l_1} G_1\, s_1\, d_1 + G_4\, s_4\, d_4} \qquad (33)$$

in which M is the cantilever moment of the center bay ($M = 0.5\, w\, l^2$); G is the elastic weight of the individual tapered members with elastic centroids O; and fixed points D, s, and d are the distances from the hinge points, respectively. No matter what cross section, shape, or irregularities the structural members have, the values G, s, and d can be accurately and reliably found as shown in Fig. 21 and outlined in the following sections.

Determination of the Elastic Weight G.—Using the author's notations, $I_A = I$, and reducing the uniform moment area $M = 1$, constant along the length of

[9] Cons. Engr., Berkeley, Calif.; Lecturer, Stanford University, Stanford, Calif.

[10] Discussion by J. J. Polivka of "An Investigation of Steel Rigid Frames," by Inge Lyse and W. E. Black, *Transactions*, ASCE, Vol. 107, 1942, p. 176.

the tapered beam, by the ratio I_x/I (the ordinates at 4 or 5 points will be sufficient to draw the reduced moment area), the centroidal axis of the reduced moment area determines point O, and the reduced moment area represents G.

Determination of Fixed Point D.—Proceed in the same manner as for the determination of G using, however, a triangular moment area with maximum ordinate = 1 as a basis. The reduced area represents the elastic rotation of the beam at the hinge due to the force producing $M = 1$ at the other end of the beam.

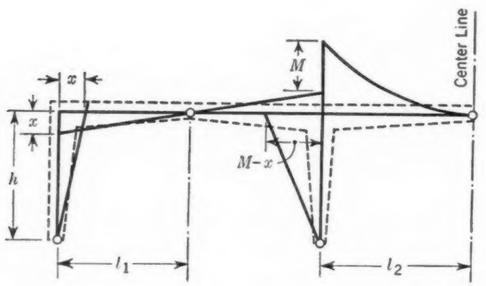


FIG. 20.—THREE-BAY FRAME WITH SYMMETRICAL FRAMING AND LOADING

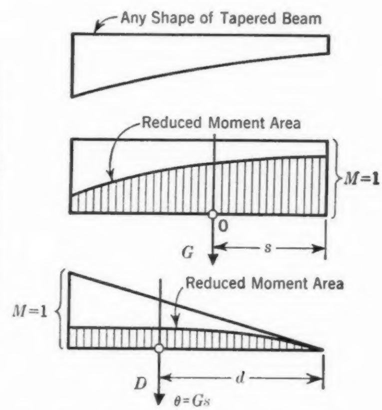


FIG. 21.—ANALYSIS OF ODD-SHAPED BEAM

The writer has demonstrated that continuous rigid frames involving any number of redundants can be solved directly without equations. Similar methods can be developed for frameworks consisting of tapered, hinged members with sufficient accuracy for economical design.

ARSHAM AMIRIKIAN,[11] M. ASCE.—The supplementary data and some of the tabulations submitted by Messrs. Ernst, Hromadik, and Swihart will unquestionably be helpful in the applications of the method. The presentation of typical hinge details in the paper was made merely as a suggestion, leaving the selection or the development of differing details to the designer's choice. Undoubtedly, there are a number of other types, including the Considère hinge, which can be used advantageously in various framing arrangements. However, in the case of hinges of that type, the desirability of the suggested utilization of the limited moment capacity is open to question because of the resulting complications in the analysis. In design practice, the omission of aid provided by such moments may be considered as an added measure of safety.

The writer fully realized that the derived mathematical expressions for k, given in Tables 1 to 5, would not be sufficient for a practical use of the method. With this thought, he also compiled extensive numerical tables and plotted curves similar to those shown in Fig. 17, covering a great number of load con-

[11] Chf. Designing Engr., Bureau of Yards and Docks, U. S. Dept. of the Navy, Washington, D. C.

ditions. Unfortunately, imposed space limitations necessitated their omission from the text. However, it is hoped, perhaps through another medium, an opportunity may be had for their eventual presentation in a future elaborated version of the paper.

The advantage in brevity obtained by the suggested combination of Tables 7, 8, and 9 into a single form, Table 10, must be weighed against the resulting loss of clarity of presentation of the various coefficients and the greater care needed in the use of the table. On the other hand, Table 11 provides considerable additional data pertaining to some of the unusual conditions of loading not completely treated in the paper.

Regarding the relative merit of the various methods of solving the simultaneous equations which result in the analysis, the writer's preference for the so-called "successive increments" method, which he has developed and used advantageously in a wide variety of design work, is predicated on the following practical considerations:

1. Compared to classical methods, it is a much simpler procedure.
2. It does not need the use of computing machines to assure a reasonable degree of accuracy, the use of an ordinary slide rule being amply adequate.
3. It provides a much greater operational flexibility, since the number of cycles or increments may vary in accordance with the desired accuracy.
4. Its accuracy is less sensitive to possible operational errors, since each successive increment is only a part of the total value, and thus an error in a given cycle would be only feebly reflected in the succeeding increments.

Mr. Sawyer has made a limited study in his attempt to appraise the economic merit of wedge-beam framing. For this purpose, a partial comparison is made with a so-called "post and lintel" framing. From the point of view of shop fabrication, it is interesting to note that the elements of framing used in the latter system do not appreciably differ from those of wedge-beam framing, since they consist of tapered sections. However, in arrangement, because of reversal of the directions of taper, two entirely dissimilar patterns of stress are obtained. Thus, the points of maximum strength in the "post and lintel" framing are at the middle of the lintels or beams and at the bottom of the posts. Obviously, such a system will be unstable under lateral loading when used in a framing more than one story high. Even in the case of a single-story framing, the lateral resistance will be solely a function of the moment capacity at the base or foundation.

As in the case of all comparisons, conclusions drawn from partial studies are of little value. Reliable comparative data can be obtained only from complete designs of actual framing. The writer had a number of opportunities of making such investigations in connection with the desired choice of a framing. In all these studies, the total material required by the wedge-beam system was found to be appreciably less than that obtained either for simple or rigid framing, the extent of material savings varying from 20% to as much as 40%.

Mr. Sawyer's suggested use of the moment distribution method in the analysis raises an interesting question for discussion. In the minds of most

engineers, this method is associated with a marvelous magic without which no procedure or solution is deemed acceptable or practicable.

In the analysis of any statically indeterminate system of framing there are a minimum number of unknowns, usually consisting of moments and reactions, which must be solved for determining the magnitude of stresses in the component members. For this purpose, it is necessary to set an equal number of relations between the unknowns and solve for their values. Regardless of what method of analysis is used, this is the work that must be accomplished—directly or indirectly. In the case of moment distribution (which, in reality, is an application of slope deflection) the required equations, consisting of the summations of moments at each joint, are set indirectly,[12] and the values of the unknowns are obtained by an arithmetical routine of solution similar to that outlined in Fig. 15. Except for the simple cases of frame analysis, such as given in the original presentation of the method,[8] the claimed advantage of operation is open to serious question. This is particularly true when the factors of interrelation (distribution and carry-over factors) cannot be readily formulated. The proposed application of the method to wedge-beam framing falls within this category. With the very simple procedure outlined in the paper, available for obtaining the equations for the redundant reactions, the writer would not choose to follow a devious procedure for attaining the same objective. A comparative solution of moments for any one of the frames used as illustrative examples in the paper by the suggested method will suffice to demonstrate this point.

Mr. Polivka suggests a semigraphical solution for frames composed of members of differing shape. In such cases, assuming also that the numerical values of k are not available or readily obtainable, a better procedure would be to utilize the well-known and simpler concept of moment area, as expressed by Eq. 17. For this purpose it will suffice to draw only one moment diagram for each member, reduce the moment ordinates by the ratios of I_x/I, obtain the moment area thus reduced, and determine its centroid. If the moments are induced by end forces only—that is, hinge reactions, which is the case of members 1, 2, 3, and 4 in Fig. 20—the lower diagram in Fig. 21, containing the values of D and d, is all that will be needed for the desired deflections and, hence, the middle or G-diagram becomes superfluous. This will be evident also from the consideration that

$$G\,s = p\,\frac{l^2}{E\,I} = D \qquad (34)$$

in which p is the reduced moment area of a cantilever of unit length under a unit load at its end.

Mr. Polivka's statement regarding the solution of redundants without equations is not clear. Assumedly this pertains to an unexplained procedure where the needed relations are formulated indirectly. However, this has not been done in his example, since Eq. 32 is the needed relation for the single redundant

[12] "Analysis of Rigid Frames," by A. Amirikian, U. S. Govt. Printing Office, Washington, D. C., 1942, p. 83.

of the frame, which equation is derived and solved in a manner similar to that given in the paper.

Gable Framing.—In the case of framing arrangements where the axes of the members deviate from a horizontal or vertical (mentioned by Mr. Sawyer), it becomes necessary to consider the effect of both vertical and horizontal reactions at the joints. As a result, the deflection equations will contain additional terms from some of these members. Except for this modification, the analysis follows the same procedure as that given for rectangular frames. The application can be illustrated by two examples.

Example 1. Symmetrical Framing.—The three-bay gable bent shown in Fig. 22(*a*) represents a simple case of symmetrical framing. Under the indicated unsymmetrical loading there are two unknown hinge reactions at joint D. The two deflection equations necessary for the solution of H and V are obtained as follows:

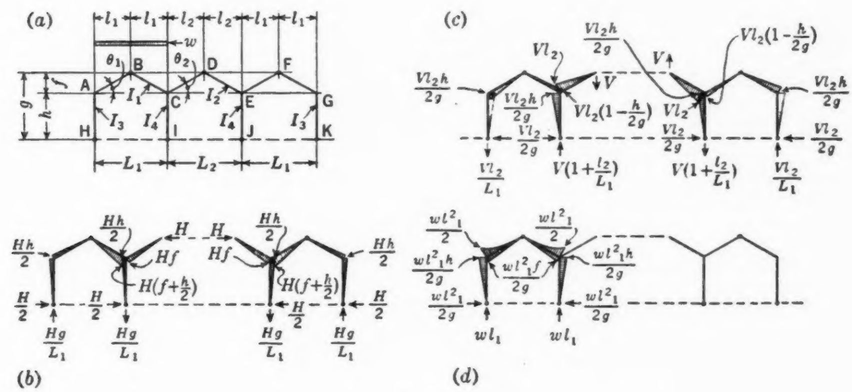


FIG. 22

To determine this horizontal deflection at hinge D

$$\Delta_{1H} = \frac{H\, l_1}{\cos\theta_1}\left[2\left(\frac{h}{2}\right)^2 + 2\left(f + \frac{h}{2}\right)^2\right]\frac{k_1}{I_1} + V \times 0$$

$$+ w\, l^2_1 \frac{l_1}{\cos\theta_1}\left[\frac{h}{2} - \left(f + \frac{h}{2}\right)\right]\frac{k_w}{I_1} - w\, l^2_1 \frac{l_1}{\cos\theta}$$

$$\times \frac{f}{2\,g}\left[\frac{h}{2} - \left(f + \frac{h}{2}\right)\right]\frac{k_1}{I_1} \ldots (35a)$$

$$\Delta_{2H} = 2\, H \frac{l_2}{\cos\theta_2} \times f^2 \times \frac{k_2}{I_2} + V \times 0 + w \times 0 \ldots\ldots\ldots (35b)$$

$$\Delta_{3H} = 2\, H\, h \left(\frac{h}{2}\right)^2 \frac{k_3}{I_3} + V \times 0 + w\, l^2_1 \frac{h^3}{4\,g} \times \frac{k_3}{I_3} \ldots\ldots\ldots (35c)$$

and

$$\Delta_{4H} = 2\,H\,h\left(\frac{h}{2}\right)^2\frac{k_4}{I_4} + V \times 0 - w\,l^2{}_1\,\frac{h^3}{4\,g} \times \frac{k_4}{I_4} \qquad (35d)$$

also

$$\Delta_{1H} + \Delta_{2H} + \Delta_{3H} + \Delta_{4H} = \Delta_H = 0 \qquad (36)$$

from which

$$H = \frac{w\,l^2{}_1\left[\frac{l_1}{\cos\theta_1} \times \frac{f}{I_1}\left(k_w - \frac{f\,k_1}{2\,g}\right) + \frac{h^3}{4\,g}\left(\frac{k_4}{I_4} - \frac{k_3}{I_3}\right)\right]}{\frac{l_1}{\cos\theta_1}(f^2 + g^2)\frac{k_1}{I_1} + \frac{2\,l_2\,f^2}{\cos\theta_2}\frac{k_2}{I_2} + \frac{h^3}{2}\left(\frac{k_3}{I_3} + \frac{k_4}{I_4}\right)} \qquad (37)$$

To determine the vertical deflection at hinge D

$$\Delta_{1V} = V\,\frac{l_1}{\cos\theta_1}\left[2\left(\frac{l_2\,h}{2\,g}\right)^2 + 2\,l^2{}_2\left(1 - \frac{h}{2\,g}\right)^2\right]\frac{k_1}{I_1} + H \times 0$$
$$+ w\,l^2{}_1\,\frac{l_1}{\cos\theta_1}\left[\frac{l_2\,h}{2\,g} - l_2\left(1 - \frac{h}{2\,g}\right)\right]\frac{k_w}{I_1} + w\,l^2{}_1\,\frac{l_1}{\cos\theta_1}$$
$$\times \frac{f}{2\,g}\left[\frac{l_2\,h}{2\,g} - l_2\left(1 - \frac{h}{2\,g}\right)\right]\frac{k_1}{I_1} \qquad (38a)$$

$$\Delta_{2V} = 2\,l^2{}_2\,\frac{l_2}{\cos\theta_2} \times \frac{k_2}{I_2} + H \times 0 + w \times 0 \qquad (38b)$$

$$\Delta_{3V} = 2\,V\,h\left(\frac{l_2\,h}{2\,g}\right)^2 \times \frac{k_3}{I_3} + H \times 0 - w\,l^2{}_1\,\frac{l_2\,h^3}{4\,g^2} \times \frac{k_3}{I_3} \qquad (38c)$$

$$\Delta_{4V} = 2\,V\,h\left(\frac{l_2\,h}{2\,g}\right)^2 \times \frac{k_4}{I_4} + H \times 0 - w\,l^2{}_1\,\frac{l_2\,h^3}{4\,g^2} \times \frac{k_4}{I_4} \qquad (38d)$$

also, similar to Eq. 36,

$$\Delta_{1V} + \Delta_{2V} + \Delta_{3V} + \Delta_{4V} = \Delta_V = 0 \qquad (39)$$

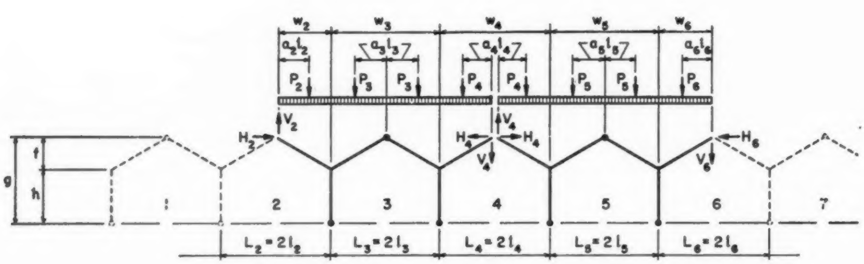


Fig. 23

TABLE 14.—CONTRIBUTION OF MEMBERS TO THE DEFLECTION OF A JOINT (JOINT 4, FIG. 23) IN INTERIOR BAYS OF GABLE FRAMES

HORIZONTAL DEFLECTION AT JOINT 4																	
MEMBER	$C=k\frac{l}{I}h$	H-V COEFFICIENTS FOR Δ_{H4}						LOAD COEFFICIENTS FOR Δ_{H4}									
		hH_2	hH_4	hH_6	l_2V_2	l_4V_4	l_6V_6	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	$(1-a_5)l_5P_5$	$(1-a_6)l_6P_6$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$	$w_5l_5^2$	$w_6l_6^2$
BEAM $3_{L,R}$	C_3	$\frac{f^2-g^2}{2h^2}$	$\frac{f^2+g^2}{2h^2}$		$\frac{f^2-g^2}{2gh}$	$-\frac{f^2+g^2}{2gh}$		$\frac{g^2-f^2}{2gh}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_3)k}$	$-\frac{f^2+g^2}{2gh}$			$\frac{g^2-f^2}{4gh}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$-\frac{f^2+g^2}{4gh}$		
$4_{L,R}$	C_4		$\frac{2f^2}{h^2}$							$-\frac{2fk_p}{h(1-a_4)k}$					$-\frac{2fk_w}{hk}$		
$5_{L,R}$	C_5		$\frac{f^2+g^2}{2h^2}$	$\frac{f^2-g^2}{2h^2}$		$\frac{f^2+g^2}{2gh}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{2gh}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_5)k}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{4gh}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$\frac{g^2-f^2}{4gh}$
COL. 2-3	$C_{2,3}$	$-\frac{1}{4}$	$\frac{1}{4}$		$\frac{h}{4g}$	$-\frac{h}{4g}$		$-\frac{h}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$-\frac{h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$		
3-4	$C_{3,4}$	$-\frac{1}{4}$	$\frac{1}{4}$		$-\frac{h}{4g}$	$\frac{h}{4g}$		$\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$		
4-5	$C_{4,5}$		$\frac{1}{4}$	$-\frac{1}{4}$		$-\frac{h}{4g}$	$\frac{h}{4g}$			$\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$
5-6	$C_{5,6}$		$\frac{1}{4}$	$-\frac{1}{4}$		$\frac{h}{4g}$	$-\frac{h}{4g}$			$-\frac{h}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$-\frac{h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$
VERTICAL DEFLECTION AT JOINT 4																	
MEMBER	$C'=k\frac{l}{I}l_4$	H-V COEFFICIENTS FOR Δ_{V4}						LOAD COEFFICIENTS FOR Δ_{V4}									
		hH_2	hH_4	hH_6	l_2V_2	l_4V_4	l_6V_6	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	$(1-a_5)l_5P_5$	$(1-a_6)l_6P_6$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$	$w_5l_5^2$	$w_6l_6^2$
BEAM $3_{L,R}$	C'_3	$\frac{g^2-f^2}{2gh}$	$-\frac{f^2+g^2}{2gh}$		$\frac{g^2-f^2}{2g^2}$	$\frac{f^2+g^2}{2g^2}$		$\frac{f^2-g^2}{2g^2}$	$\frac{fk_p}{g(1-a_3)k}-\frac{f^2}{g^2}$	$\frac{f^2+g^2}{2g^2}$			$\frac{f^2-g^2}{4g^2}$	$\frac{fk_w}{gk}-\frac{f^2}{2g^2}$	$\frac{f^2+g^2}{4g^2}$		
$4_{L,R}$	C'_4					2											
$5_{L,R}$	C'_5		$\frac{f^2+g^2}{2gh}$	$\frac{f^2-g^2}{2gh}$		$\frac{f^2+g^2}{2g^2}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{2g^2}$	$\frac{f^2}{g^2}-\frac{fk_p}{g(1-a_5)k}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{4g^2}$	$\frac{f^2}{2g^2}-\frac{fk_w}{gk}$	$\frac{g^2-f^2}{4g^2}$
COL. 2-3	$C'_{2,3}$	$\frac{h}{4g}$	$-\frac{h}{4g}$		$-\frac{h^2}{4g^2}$	$\frac{h^2}{4g^2}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$\frac{h^2}{8g^2}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
3-4	$C'_{3,4}$	$-\frac{h}{4g}$	$\frac{h}{4g}$		$-\frac{h^2}{4g^2}$	$\frac{h^2}{4g^2}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$\frac{h^2}{8g^2}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
4-5	$C'_{4,5}$		$-\frac{h}{4g}$	$\frac{h}{4g}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$
5-6	$C'_{5,6}$		$\frac{h}{4g}$	$-\frac{h}{4g}$		$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$

from which

$$V = \frac{w\,l^2_1 \left[\frac{l_1}{\cos\theta_1} \times \frac{f}{g\,I_1}\left(k_w + \frac{f}{2\,g}\,k_1\right) + \frac{h^3}{4\,g^2}\left(\frac{k_3}{I_3} + \frac{k_4}{I_4}\right)\right]}{l_2 \left[\frac{l_1}{\cos\theta_1}\left(\frac{f^2+g^2}{g^2}\right)\frac{k_1}{I_1} + \frac{2\,l_2}{\cos\theta_2} \times \frac{k_2}{I_2} + \frac{h^3}{2\,g^2}\left(\frac{k_3}{I_3} + \frac{k_4}{I_4}\right)\right]} \quad \ldots (40)$$

As in the case of rectangular framing, the deflection equations can be obtained directly from Eqs. 16. The data necessary for the determination of the various coefficients of those equations are given in Tables 14 (with Fig. 23), 15 (with Fig. 24), and 16.

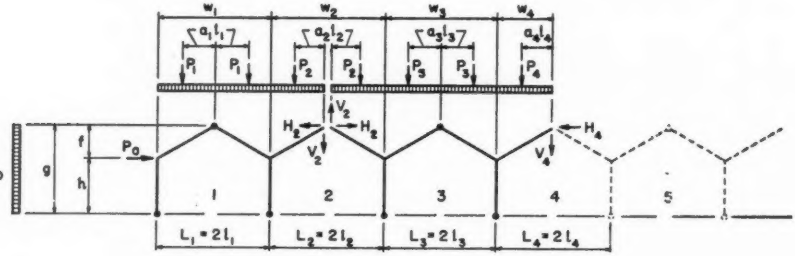


FIG. 24

Example 2. Unsymmetrical Framing.—The three-bay bent shown in Fig. 25(*a*) illustrates this case. Here, as a result of the dissymmetry, in writing the deflection expressions, the two roof beams in each bay are no longer considered in pairs but are treated separately, as independent members, similar to

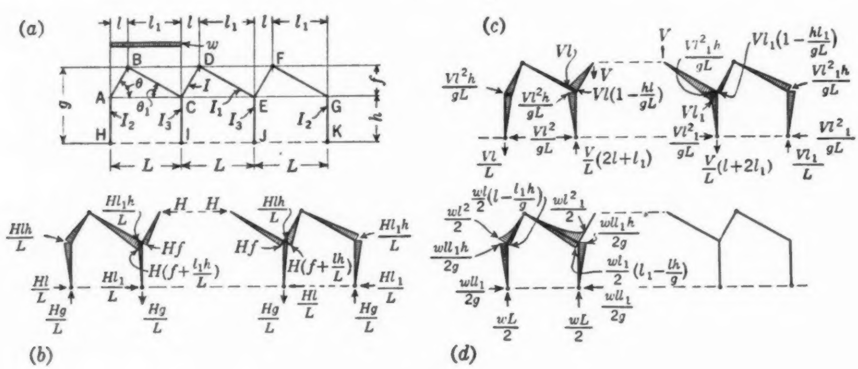


FIG. 25

the case in which the members are of differing shape or where the loading in a bay is unsymmetrically applied. The equilibrium diagrams for H and V (the two unknown reactions at hinge D) and for the loading in the first bay, obtained as simple-frame moments, are given in Fig. 25(*b*) to Fig. 25(*d*), respec-

TABLE 15.—CONTRIBUTION OF MEMBERS TO THE DEFLECTION OF A JOINT (JOINT 2, FIG. 24) IN END BAYS OF GABLE FRAMES WITH AN ODD NUMBER OF BAYS

HORIZONTAL DEFLECTION AT JOINT 2															
MEMBER	$C=k\frac{l}{I}h$	H-V COEFFICIENTS FOR Δ_{H2}				LOAD COEFFICIENTS FOR Δ_{H2}									
		hH_2	hH_4	l_2V_2	l_4V_4	hP_0	$(1-a_1)l_1P_1$	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	w_0g^2	$w_1l_1^2$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$
BEAM $1_{L,R}$	C_1	$\frac{f^2+g^2}{2h^2}$		$-\frac{f^2+g^2}{2gh}$		$-\frac{f+g}{2g}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_1)k}$	$-\frac{f^2+g^2}{2gh}$			$\frac{flk_w}{2g^2k}-\frac{f+g}{4g}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$-\frac{f^2+g^2}{4gh}$		
$2_{L,R}$	C_2	$\frac{2f^2}{h^2}$						$-\frac{2fk_p}{h(1-a_2)k}$					$-\frac{2fk_w}{hk}$		
$3_{L,R}$	C_3	$\frac{f^2+g^2}{2h^2}$	$\frac{f^2-g^2}{2h^2}$	$\frac{f^2+g^2}{2gh}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{2gh}$	$\frac{f^2}{gh}-\frac{fk_p}{h(1-a_3)k}$	$\frac{g^2-f^2}{2gh}$			$-\frac{f^2+g^2}{4gh}$	$\frac{f^2}{2gh}-\frac{fk_w}{hk}$	$\frac{g^2-f^2}{4gh}$
COL. 0-1	$C_{0,1}$	$\frac{1}{4}$		$-\frac{h}{4g}$		$-\frac{f+g}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$\frac{h^2k_w}{2g^2k}-\frac{3h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$		
1-2	$C_{1,2}$	$\frac{1}{4}$		$\frac{h}{4g}$		$-\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$-\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$		
2-3	$C_{2,3}$	$\frac{1}{4}$	$-\frac{1}{4}$	$-\frac{h}{4g}$	$\frac{h}{4g}$			$\frac{h}{4g}$	$-\frac{h}{2g}$	$\frac{h}{4g}$			$\frac{h}{8g}$	$-\frac{h}{4g}$	$\frac{h}{8g}$
3-4	$C_{3,4}$	$\frac{1}{4}$	$-\frac{1}{4}$	$\frac{h}{4g}$	$-\frac{h}{4g}$			$-\frac{h}{4g}$	$\frac{h}{2g}$	$-\frac{h}{4g}$			$-\frac{h}{8g}$	$\frac{h}{4g}$	$-\frac{h}{8g}$
VERTICAL DEFLECTION AT JOINT 2															
MEMBER	$C'=k\frac{l}{I}l_2$	H-V COEFFICIENTS FOR Δ_{V2}				LOAD COEFFICIENTS FOR Δ_{V2}									
		hH_2	hH_4	l_2V_2	l_4V_4	hP_0	$(1-a_1)l_1P_1$	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	$(1-a_4)l_4P_4$	w_0h^2	$w_1l_1^2$	$w_2l_2^2$	$w_3l_3^2$	$w_4l_4^2$
BEAM $1_{L,R}$	C'_1	$-\frac{f^2+g^2}{2gh}$		$\frac{f^2+g^2}{2g^2}$		$\frac{g^2-f^2}{2g^2}$	$\frac{fk_p}{g(1-a_1)k}-\frac{f^2}{g^2}$	$\frac{f^2+g^2}{2g^2}$			$\frac{g^2-f^2}{4h^2}-\frac{flk_w}{2ghk}$	$\frac{fk_w}{gk}-\frac{f^2}{2g^2}$	$\frac{f^2+g^2}{4g^2}$		
$2_{L,R}$	C'_2			2											
$3_{L,R}$	C'_3	$\frac{f^2+g^2}{2gh}$	$\frac{f^2-g^2}{2gh}$	$\frac{f^2+g^2}{2g^2}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{2g^2}$	$\frac{f^2}{g^2}-\frac{fk_p}{g(1-a_3)k}$	$\frac{g^2-f^2}{2g^2}$			$-\frac{f^2+g^2}{4g^2}$	$\frac{f^2}{2g^2}-\frac{fk_w}{gk}$	$\frac{g^2-f^2}{4g^2}$
COL. 0-1	$C'_{0,1}$	$-\frac{h}{4g}$		$\frac{h^2}{4g^2}$		$\frac{g^2-f^2}{4g^2}$	$\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$\frac{3}{8}-\frac{hk_w}{2gk}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
1-2	$C'_{1,2}$	$\frac{h}{4g}$		$\frac{h^2}{4g^2}$		$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$\frac{h^2}{4g^2}$			$-\frac{1}{8}$	$-\frac{h^2}{4g^2}$	$\frac{h^2}{8g^2}$		
2-3	$C'_{2,3}$	$-\frac{h}{4g}$	$\frac{h}{4g}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$
3-4	$C'_{3,4}$	$\frac{h}{4g}$	$-\frac{h}{4g}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{4g^2}$	$\frac{h^2}{2g^2}$	$-\frac{h^2}{4g^2}$			$-\frac{h^2}{8g^2}$	$\frac{h^2}{4g^2}$	$-\frac{h^2}{8g^2}$

tively. Then, from the consideration of horizontal deflection at point D,

$$\Delta_H = \frac{H\,l}{\cos\theta} \times \frac{k}{I}\left[\left(\frac{l\,h}{L}\right)^2 + f^2 + \left(f + \frac{l\,h}{L}\right)^2\right] + \frac{H\,l_1}{\cos\theta_1} \times \frac{k_1}{I_1}\left[\left(\frac{l_1\,h}{L}\right)^2\right.$$

$$\left. + f^2 + \left(f + \frac{l_1\,h}{L}\right)^2\right] + H\,h^3\frac{k_2}{I_2}\left[\frac{l^2}{L^2} + \frac{l^2_1}{L^2}\right] + H\,h^3\frac{k_3}{I_3}\left[\frac{l^2}{L^2} + \frac{l^2_1}{L^2}\right]$$

$$- \frac{V\,l}{\cos\theta} \times \frac{k}{I}\left[\frac{l^3\,h^2}{L^2\,g} + f\,l - \left(f + \frac{l\,h}{L}\right)\left(l_1 - \frac{h\,l^2_1}{L\,g}\right)\right] + \frac{V\,l_1}{\cos\theta_1}$$

$$\times \frac{k_1}{I_1}\left[f\,l_1 - \left(f + \frac{h\,l_1}{L}\right)\left(l - \frac{h\,l^2}{L\,g}\right) + \frac{h^2\,l_1^3}{L^2\,g}\right]$$

$$+ V\,h^3\frac{k_2}{I_2}\left[\frac{l^3_1}{L^2\,g} - \frac{l^3}{L^2\,g}\right] + V\,h^3\frac{k_3}{I_3}\left[\frac{l_1\,l^2}{L^2\,g} - \frac{l^2_1\,l}{L^2\,g}\right]$$

$$+ \frac{w\,l^3}{\cos\theta} \times \frac{k_w}{I}\left[\frac{h\,l}{L}\right] - \frac{w\,l^3_1}{\cos\theta_1} \times \frac{k_{w1}}{I_1}\left[\left(f + \frac{h\,l_1}{L}\right)\right]$$

$$+ w\,h^3\frac{k_2}{I_2}\left[\frac{l^2\,l_1}{2\,L\,g} - w\,h^3\frac{k_3}{I_3}\,\frac{l\,l^2_1}{2\,L\,g}\right] - \frac{w\,l}{\cos\theta} \times \frac{k}{I}\left[\frac{h\,l^2}{2\,L}\left(l - \frac{h\,l_1}{g}\right)\right]$$

$$+ \frac{w\,l_1}{2\cos\theta_1} \times \frac{k_1}{I_1}\left[\left(f + \frac{h\,l_1}{L}\right)\left(l^2_1 - \frac{h\,l\,l_1}{g}\right)\right] = 0 \qquad (41)$$

Similarly, consideration of the vertical deflection at the same joint will yield the following relation:

$$\Delta_V = \frac{V\,l}{\cos\theta} \times \frac{k}{I}\left[\left(\frac{h\,l^2}{g\,L}\right)^2 + l^2 + l^2_1\left(1 - \frac{h\,l_1}{g\,L}\right)^2\right] + \frac{V\,l_1}{\cos\theta_1} \times \frac{k_1}{I_1}\left[\left(\frac{h\,l^2_1}{g\,L}\right)^2\right.$$

$$\left. + l^2_1 + l^2\left(1 - \frac{h\,l}{g\,L}\right)^2\right] + V\,h^3\frac{k_2}{I_2}\left[\left(\frac{l^2}{g\,L}\right)^2 + \left(\frac{l^2_1}{g\,L}\right)^2\right]$$

$$+ V\,h^3\frac{k_3}{I_3}\left[\left(\frac{l^2}{g\,L}\right)^2 + \left(\frac{l^2_1}{g\,L}\right)^2\right] - \frac{H\,l}{\cos\theta}$$

$$\times \frac{k}{I}\left[\frac{h^2\,l^3}{g\,L^2} + f\,l - l_1\left(1 - \frac{h\,l_1}{g\,L}\right)\left(f + \frac{h\,l}{L}\right)\right] - \frac{H\,l_1}{\cos\theta_1}$$

$$\times \frac{k_1}{I_1}\left[l\left(1 - \frac{h\,l}{g\,L}\right)\left(f + \frac{h\,l_1}{L}\right) - f\,l_1 - \frac{h^2\,l^3_1}{g\,L^2}\right]$$

$$- H\,h^3\frac{k_2}{I_2}\left[\frac{l^3}{g\,L^2} - \frac{l^3_1}{g\,L^2}\right] - H\,h^3\frac{k_3}{I_3}\left[\frac{l\,l^2_1}{g\,L^2} - \frac{l_1\,l^2}{g\,L^2}\right]$$

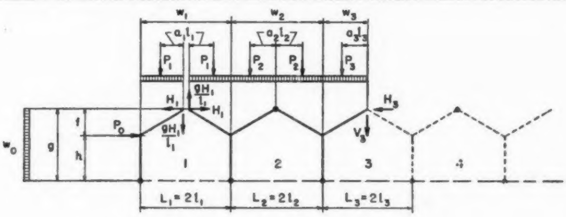
$$- w\,\frac{k_w}{I} \times \frac{h\,l^2}{g\,L} \times \frac{l^3}{\cos\theta} + w\,\frac{k}{I} \times \frac{h\,l^2}{g\,L} \times \frac{l}{2}\left(l - \frac{h\,l_1}{g}\right)$$

$$\times \frac{l}{\cos\theta} + w\,\frac{k_{w\,l}}{I_1} \times l\left(1 - \frac{h\,l}{g\,L}\right)\frac{l^3_1}{\cos\theta_1} - w\,\frac{k_1}{I_1}$$

$$\times l\left(1 - \frac{h\,l}{g\,L}\right)\frac{l_1}{2}\left(l_1 - \frac{h\,l}{g}\right)\frac{l_1}{\cos\theta_1} - w\,\frac{k_2}{I_2}$$

$$\times \frac{l^3\,l_1}{2\,g^2\,L}\,h^3 - w\,\frac{k_3}{I_3} \times \frac{l^3\,l_1}{2\,g^2\,L}\,h^3 = 0 \qquad (42)$$

TABLE 16.—CONTRIBUTION OF MEMBERS TO THE DEFLECTION OF A JOINT IN END BAYS OF GABLE FRAMES WITH AN EVEN NUMBER OF BAYS (HORIZONTAL DEFLECTION AT JOINT 1)



HORIZONTAL DEFLECTION AT JOINT 1												
MEMBER	$C = k\frac{1}{I}h$	H-V COEF'S FOR Δ_{01}			LOAD COEFFICIENTS FOR Δ_{01}							
		hH_1	hH_3	l_3V_3	hP_0	$(1-a_1)l_1P_1$	$(1-a_2)l_2P_2$	$(1-a_3)l_3P_3$	w_0g^2	$w_1l_1^2$	$w_2l_2^2$	$w_3l_3^2$
BEAM $1_{L,R}$	C_1	$\frac{2(f^2+g^2)}{h^2}$			$-\frac{2g}{h}$	$\frac{2f^2}{gh}-\frac{2fk_p}{h(1-a_1)k}$			$\frac{flk_w}{g^2k}-\frac{g}{h}$	$\frac{f^2}{gh}-\frac{2fk_w}{hk}$		
$2_{L,R}$	C_2	$\frac{2(f^2+g^2)}{h^2}$	$\frac{f^2-g^2}{h^2}$	$\frac{f+g}{g}$	$-\frac{f^2+g^2}{gh}$		$\frac{2f^2}{gh}-\frac{2fk_p}{h(1-a_2)k}$	$\frac{f+g}{g}$	$-\frac{f^2+g^2}{2gh}$		$\frac{f^2}{gh}-\frac{2fk_w}{hk}$	$\frac{f+g}{2g}$
$3_{L,R}$	C_3											
COL. 0-1	$C_{0,1}$	1			-1	$\frac{h}{g}$			$\frac{h^2k_w}{g^2k}-\frac{h}{g}$	$\frac{h}{2g}$		
1-2	$C_{1,2}$											
2-3	$C_{2,3}$	1	$-\frac{1}{2}$	$-\frac{h}{2g}$	$-\frac{h}{2g}$		$\frac{h}{g}$	$-\frac{h}{2g}$	$-\frac{h}{4g}$		$\frac{h}{2g}$	$-\frac{h}{4g}$

Corrections for Transactions.—In Fig. 13(*a*) change the seven lower dimensions "l" to "L"; in Fig. 14(*a*), "$a_1 l_1$" is the eccentricity of load P; at the two vertical reactions in Fig. 14(*a*) change $\frac{\text{“}P\,l_1\text{”}}{2}$ to $\frac{\text{“}P\text{”}}{2}$; at the right-hand vertical reaction in Fig. 14(*c*) change "$w\,l_1$" to "$w\,l_2$"; in the heading of Col. 10, Table 6, change "t_{n-1}" to "P_{n-1}"; on page 25, in line 4 below Eq. 16*b*, change "Table 7(a)" to "Table 6(b)"; in Eqs. 17, change the formula for Q_n as follows: "$Q_n = w_{n+1}\, l^2{}_{n+1}\, [C_6\, t_{nw(n+1)6} + C_7\, t_{nw(n+1)7}] + (1 - a)\, l_{n-1}\, P_{n-1}\, [C_1\, t_{n(n-1)1} + C_4\, T_{n(n-1)4} + C_5\, t_{n(n-1)5}]$"; in Col. 2, Table 7 (continued) and in Col. 2, Table 8 (continued), change the seven C-symbols to C'-symbols; in the five unnumbered formulas preceding Eqs. 18, change fourteen (1 V)-symbols to (l V)-symbols; in Eqs. 18, change the three (41 V)-symbols to (4 l V)-symbols; and, in the heading of Col. 12, Table 8, and Table 8 (continued), change "$w_o\, g^2$" to "$w_o\, h^2$."

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1952

PRESIDENT

CARLTON S. PROCTOR

VICE-PRESIDENTS

Term expires October, 1952:

WILLIAM R. GLIDDEN
DANIEL V. TERRELL

Term expires October, 1953:

GEORGE W. BURPEE
A M RAWN

DIRECTORS

Term expires October, 1952:

MILTON T. WILSON
MORRIS GOODKIND

Term expires January, 1953:

OTTO HOLDEN
FRANK L. WEAVER
GORDON H. BUTLER
G. BROOKS EARNEST
GEORGE W. LAMB
EDWARD C. DOHM

Term expires October, 1953:

KIRBY SMITH
FRANCIS S. FRIEL
WALLACE L. CHADWICK
NORMAN R. MOORE
BURTON G. DWYRE
LOUIS R. HOWSON

Term expires October, 1954:

WALTER D. BINGER
FRANK A. MARSTON
GEORGE W. McALPIN
JAMES A. HIGGS
I. C. STEELE

PAST-PRESIDENTS

Members of the Board

ERNEST E. HOWARD
GAIL A. HATHAWAY

TREASURER

CHARLES E. TROUT

EXECUTIVE SECRETARY

WILLIAM N. CAREY

ASSISTANT TREASURER

GEORGE W. BURPEE

ASSISTANT SECRETARY

E. L. CHANDLER

PROCEEDINGS OF THE SOCIETY

SYDNEY WILMOT
Manager of Technical Publications

HAROLD T. LARSEN
Editor of Technical Publications

COMMITTEE ON PUBLICATIONS

LOUIS R. HOWSON

FRANCIS S. FRIEL
I. C. STEELE
OTTO HOLDEN
FRANK A. MARSTON
NORMAN R. MOORE